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Combining Echo Cancellation and Decision Feedback Equalization

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A system for two-wire, full-duplex data transmission is proposed. It consists of two adaptive transversal filters, one accepting the transmitted symbols and working as an echo canceller, the other accepting the received symbols and functioning as a decision feedback equalizer. A joint stochastic adjustment algorithm (updates at each baud) is analyzed, and it is shown that the sum of the mean-squared errors in the coefficients of both filters can be decoupled from its difference by selecting identical gain constants in each loop. The optimum gain equals the reciprocal of the sum of the taps of both loops. Convergence is exponential, and its time is 0.23 adjustments/dB/tap. This is completely independent of all channel parameters. Implementation of the proposed structure requires neither multipliers nor A/D converters. Promising applications are seen in channels with moderate precursor distortion, such as highpass channels (dc-restoration), two-wire PBX systems with a need for high-speed, full-duplex communication, limited distance cable channels, and, most important, two-wire digital subscriber lines for digital voice/data terminals.

I. INTRODUCTION

In a previous publication, a new approach to adaptive echo cancelling for full-duplex data transmission over two-wire facilities was presented.[1] Its novelty was that the compensation signal is synthesized directly from the data symbols, rather than from the transmitter output signal, and canceller adjustments are controlled by the receiver's estimated error signal, rather than the receiver's input signal, as has been done in previous echo cancellers.[2–5] This approach can be applied as long as the underlying modulation concept is linear; it allows for considerable economies in circuit implementation and also eliminates the double talker problem. The number of taps can be kept

minimal if echo compensation is done at the baud rate, in synchronism with the receiver sampling operation. For this case, it has been shown in Ref. 1 that rapid convergence (time proportional to the number of echo taps) can be achieved, and that this convergence does not depend on channel response, echo response, timing phase, carrier phase, or the energy ratio of the echo signal to the distant received signal. Further studies dealing with this scheme are presented in Refs. 6 and 7.

On many real channels, the echo canceller alone would solve only part of the problem, since intersymbol interference (ISI) is severe and must be properly dealt with. The well-known adaptive equalizer is the proper cure for this, and during the past decade its art has been refined to a level of high sophistication. However, for two-wire full-duplex communication, one now must in general deal both with an adaptable echo canceller *and* an equalizer. Their joint adaptive adjustment will create new problems as far as updating techniques and dynamic behavior are concerned. Preliminary investigations of the behavior of a linear equalizer and an echo canceller have been carried out by Falconer and Weinstein,[8] indicating that convergence critically depends on the received signal to echo power ratio. These results have also been summarized in Ref. 6. Undoubtedly, this is a field where further studies are essential.

In this paper, a new system is proposed which combines both adaptive echo cancellation and equalization but retains the properties of rapid, channel-independent convergence under joint adjustments. As will be seen, the equalizer has to be somewhat restricted to obtain these advantages. The architecture of this system is discussed in the next section. After this, the convergence behavior is discussed, and finally simulation runs are presented which confirm the previously established analytical results.

II. SYSTEM ARCHITECTURE AND DEFINITIONS

The basic arrangement of the proposed system is shown in Fig. 1. We concentrate on a baseband system, not because of any such limitations (all linear modulation schemes can be represented in equivalent baseband), but rather to keep notation simple and concentrate on the essentials. The system contains two adaptive transversal filters; one is connected to the transmit data symbols and the other is connected to the received data symbols. The first works as an echo canceller as proposed in Ref. 1 to mitigate the effects of hybrid mismatch; the second filter is a decision feedback equalizer which compensates for intersymbol interference in the received far end signal due to linear distortion on the channel. With the structure in Fig. 1, this compensation is limited to trailing distortion components (postcursors) and we say more about this shortly. The outputs of both filters

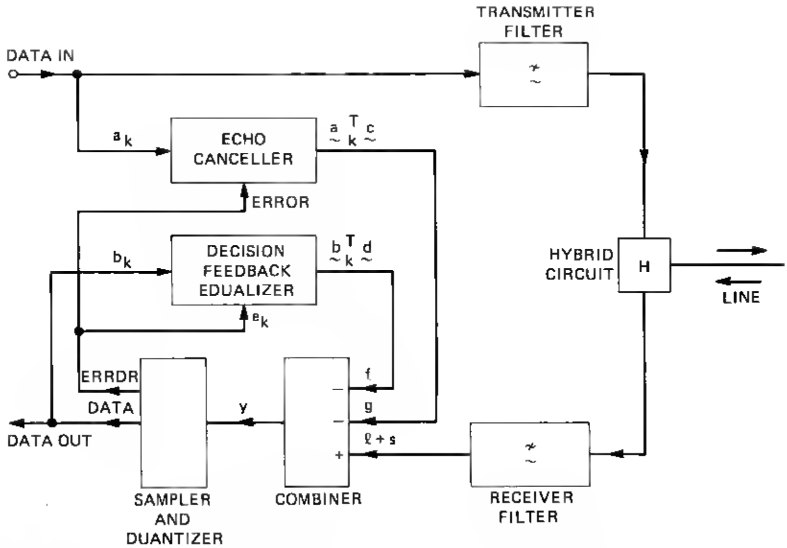

Fig. 1—Block diagram of data set with combined echo canceller and decision feedback equalizer.

are subtracted from the received signal and the resulting "cleaned-up" waveform is sampled to yield estimates $\hat{b}_k$ of the far end data b_k. Error samples e_k are generated in the usual way and are used as a common control signal to adjust both the canceller and the equalizer.

Since the equalizer has no linear taps to compensate for precursors, its abilities are somewhat limited. However, several significant advantages are also obtained: the implementation is economic since no A/D converter is required and the usually painful multiplications are replaced by simple additions (at least for binary and pseudo-ternary signals; some other codes may require multiplications where one factor is a two- or three-bit number). A further advantage is the total stability and predictable performance of this system, as is apparent from the analysis presented in Section III. Decision feedback equalization alone is well suited for highpass channels requiring dc restoration. It will also have interesting applications in transmission over cables and other channels where the distortion is predominantly of the trailing type.

In accordance with Ref. 1, let the near-end and far-end symbols be statistically and mutually independent random variables a_k and b_k. With h_k and r_k, we denote the samples of the channel response and echo response. At $t_0 + kT$, the received signal consists of a far-end component

$$\ell_k = \sum_{i=-\infty}^{\infty} b_{k-i} h_i \tag{1}$$

and an echo signal

$$s_k = \sum_{i=-N}^{\infty} a_{k-i} r_i. \tag{2}$$

The receiver in Fig. 1 synthesizes the two compensation signals

$$g_k = \sum_{i=-N}^{M} a_{k-i} c_i = \mathbf{a}_k^T \mathbf{c} \tag{3}$$

and

$$f_k = \sum_{i=1}^{J} b_{k-i} d_i = \mathbf{b}_k^T \mathbf{d}, \tag{4}$$

where (3) is formed by an echo canceller with $L = M + N + 1$ taps $c_{-N} \cdots c_M$ and (4) is the output of the decision feedback equalizer* comprising taps $d_1 \cdots d_J$. The combined output y is

$$y_k = \ell_k + s_k - f_k - g_k + \zeta_k, \tag{5}$$

where ζ is some additive channel noise with variance σ^2. The error signal becomes

$$\begin{aligned} e_k &= y_k - b_k \\ &= \sum_{i=1}^{J} b_{k-i}(h_i - d_i) + \sum_{i=-N}^{M} a_{k-i}(r_i - c_i) + w_k, \end{aligned} \tag{6}$$

where

$$\begin{aligned} w_k = \zeta_k &+ \sum_{i=-\infty}^{-1} b_{k-i} h_i + \sum_{i=J+1}^{\infty} b_{k-i} h_i \\ &+ \sum_{i=M+1}^{\infty} a_{k-i} r_i + b_k(h_0 - 1). \end{aligned} \tag{7}$$

One recognizes that w_k is the remaining error component after optimum settings for both the canceller and the equalizer have been obtained. These optimum settings are, of course, given by

$$c_i = r_i \quad i = -N \cdots M \tag{8}$$

$$d_i = h_i \quad i = 1 \cdots J, \tag{9}$$

where all echo and intersymbol interference components within the reach of the adaptive structures are fully cancelled. It is further clear that some kind of automatic gain control should be used to force h_0

* We make the usual assumption that correct decisions are entered into the equalizer.

= 1 to minimize the variance of (7). In accordance with (8) and (9), we introduce error vectors

$$\phi = \mathbf{c} - \mathbf{r} \tag{10}$$

$$\psi = \mathbf{d} - \mathbf{h} \tag{11}$$

for the canceller and equalizer coefficients. The error can now be expressed in the simple form

$$e_k = w_k - \mathbf{b}_k^T\psi - \mathbf{a}_k^T\phi. \tag{12}$$

It will be our goal to adaptively minimize the mean-square error (MSE) which is given as

$$E\{e_k^2\} = \psi^T\psi + \phi^T\phi + R, \tag{13}$$

where R denotes that part of the MSE which cannot be further reduced with the canceller/equalizer combination, i.e.,

$$R = E\{w_k^2\} = \sum_{i=-\infty}^{-1} h_i^2 + \sum_{i=J+1}^{\infty} h_i^2 + \sum_{i=M+1}^{\infty} r_i^2 + \sigma^2 + (h_0 - 1)^2, \tag{14}$$

and the first two terms in (13) represent the excess error due to misadjustment. We now investigate how this excess error can be minimized.

III. JOINT STOCHASTIC ADJUSTMENTS

A joint, stochastic updating algorithm of the form

$$\mathbf{c}_{n+1} = \mathbf{c}_n + \gamma e_n \mathbf{a}_n \tag{15}$$

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \beta e_n \mathbf{b}_n \tag{16}$$

with constant step sizes γ and β is proposed; i.e., no averaging is used. Together with (12) one obtains the coupled recursions

$$\phi_{k+1} = (I - \gamma \mathbf{a}_k \mathbf{a}_k^T)\phi_k - \gamma \mathbf{a}_k \mathbf{b}_k^T \psi_k + \gamma w_k \mathbf{a}_k \tag{17}$$

$$\psi_{k+1} = (I - \beta \mathbf{b}_k \mathbf{b}_k^T)\, \psi_k - \gamma \mathbf{b}_k \mathbf{a}_k^T \sigma_k + \beta w_k \mathbf{b}_k, \tag{18}$$

which demonstrate that adjustments in the two loops are not independent of each other. Both ϕ_k and ψ_k and therefore the excess error ϵ are of course influenced by the past history of the data symbols. Defining

$$q_k = E\{\phi_k^T\phi_k\} \tag{19}$$

$$p_k = E\{\psi_k^T\psi_k\} \tag{20}$$

and applying the independence assumptions discussed in Ref. 1, after

some manipulations one obtains

$$q_{k+1} + p_{k+1} = (1 - 2\gamma + \gamma^2 L + \beta^2 J)q_k + (1 - 2\beta + \beta^2 J + \gamma^2 L)p_k + (\gamma^2 L + \beta^2 J)R \quad (21)$$

or, after introducing

$$\epsilon_k = p_k + q_k \quad (22)$$

$$\delta_k = p_k - q_k, \quad (23)$$

this can be written as

$$\epsilon_{k+1} = (1 - \beta - \gamma + \beta^2 J + \gamma^2 L)\epsilon_k + (\gamma - \beta)\delta_k + R(\beta^2 J + \gamma^2 L). \quad (24)$$

Our only interest is to minimize the combined MSE stemming from both echoes and intersymbol interference, i.e., ϵ; we are not concerned with the behavior of either component alone. A simple recursion which depends only on the excess error ϵ alone results if we set

$$\gamma = \beta, \quad (25)$$

i.e., if equal gains are used in the echo canceller and the decision feedback equalizer loop. Note, however, that $\gamma = \beta$ will *not* eliminate the coupling between the two loops; but then this has never been our concern since our objective is to minimize the total error without regard to the convergence behavior of its components. An illustration of what this practically means will be presented in the next section.

With the difference term δ_k disappearing, the recursion (24) can easily be solved,

$$\epsilon_k = \epsilon_\infty + [1 - 2\beta + \beta^2(L + J)]^k(\epsilon_0 - \epsilon_\infty), \quad (26)$$

where ϵ_0 is the initial mean-square excess error, and

$$\epsilon_\infty = \frac{\beta(J + L)R}{2 - \beta(J + L)} \quad (27)$$

is the steady-state mean-square excess error (tap fluctuation noise) in the tracking mode. During the training mode, ϵ converges exponentially until it reaches ϵ_∞. Fastest convergence will occur if

$$\beta = \beta_{\text{opt}} = \frac{1}{J + L}, \quad (28)$$

in which case

$$\epsilon_\infty(\beta_{\text{opt}}) = R. \quad (29)$$

A comparison with Ref. 1 shows that the results regarding the echo canceller alone and those for the combination of the echo canceller and the decision feedback equalizer are related; one need only replace

the number of echo canceller taps with the sum of the taps of both the canceller and the equalizer.

Convergence with $\beta = \beta_{\rm opt}$ is governed by

$$\epsilon_k = R + \left(1 - \frac{1}{J+L}\right)^k (\epsilon_0 - R) \tag{30}$$

and during the training phase the excess mean-square error is thus reduced at an average rate of

$$\frac{4.343}{J+L} \text{ dB/adjustment,} \tag{31}$$

and convergence time is

$$0.230 \text{ adjustments/dB/tap.} \tag{32}$$

Both the above results assume $J + L \gg 1$ to linearize the logarithm in (30).

IV. SIMULATION RESULTS

The simplicity of the result obtained in the previous section speaks for itself. Most important, convergence is only determined by the sum of the taps; in particular, the ratio of echo signal power to received signal power is immaterial. Nothing could be more desirable for an actual system which is subjected to a wide variety of channel conditions.

As an example, consider a system with 127 echo canceller taps and 31 decision feedback taps. Samples of the echo response and the channel response have been taken at random. The combined mean-square error is about 100 times stronger than the received signal and consists mainly of echo noise; the ratio of echo/signal/ISI power being 100/1/1. Convergence of the total excess error ϵ is shown in Fig. 2. Note that convergence in the simulated system is somewhat faster than predicted by theory. This is because (30) was obtained as an average over *all* possible data sequences, whereas the simulation made use of maximum length pseudorandom sequences which have ideal spectral properties (and are also efficient to store). In the analysis, averaging includes such nonconverging patterns as steady mark or space. The analytical results of this and all similarly related problems tend therefore to be on the pessimistic side as far as they relate to the real world where pseudorandom training patterns are commonly used.

Our objective has always been to minimize the total noise power from all sources without caring about the reduction of the individual components. However, it is interesting to observe how each of the components p_k and q_k behaves separately during the joint training. To make this case more clear, an equalizer with $J = L = 8$ is selected, and

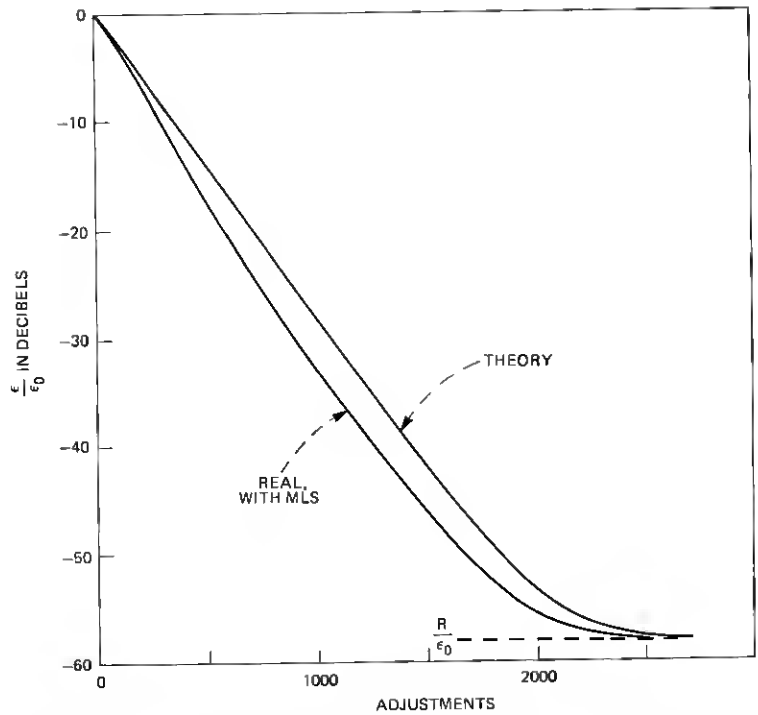

Fig. 2—Reduction of excess error in a system with 127 canceller taps and 31 decision feedback taps.

we consider various ratios of echo/signal/ISI power. The number of taps in this example may be in the order of what one would consider for two-wire full-duplex baseband transmission over limited distance cable facilities. Figure 3 depicts what is happening to the individual components p_k and q_k for echo-to-ISI power ratios r of 100, 1, and 0.01. The number of adjustments is written as a parameter along each curve. In the case where impairments due to echo noise and ISI are equal, they are reduced at about the same rate. However, if one component initially dominates, this component is reduced first, and this may actually perturb the tapsettings for the other (weaker) component in such a way as to increase its contribution temporarily until both components have been reduced to about a comparable level. From there on, they are jointly reduced at the same rate.

V. CONCLUSIONS AND SUMMARY

A combined structure incorporating an echo canceller and a decision feedback equalizer has been proposed. The structure has some appealing symmetries which can probably be exploited to realize efficient signal processing, in particular for the special case analyzed in this

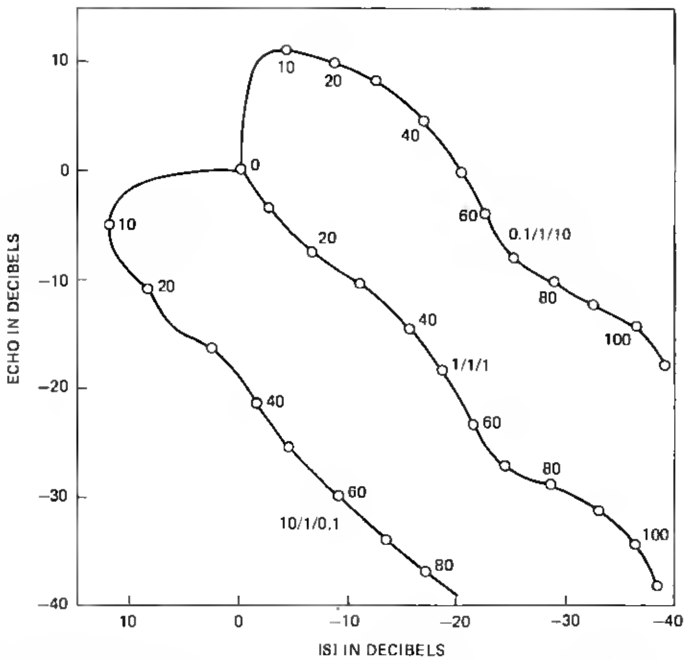

Fig. 3—Convergence of p and q components versus number of adjustments (bauds) for three ratios of echo/signal/ISI power.

paper where it has been assumed that the two communicating stations are mutually synchronized to a common master clock,* that taps are spaced at symbol intervals T, and that adjustments occur at each baud (no averaging). It has been shown that a direct, linear, first-order recursion can be obtained for the *total* excess error stemming from both the canceller and the decision feedback equalizer, provided that equal gain factors are selected for both loops. The optimum gain (in the sense of fastest convergence) equals the reciprocal of the sum of the number of taps in the two loops. Convergence is exponential, and the number of bauds required to obtain a certain improvement is 0.23 baud/dB/tap. Using the optimum gain results in a 3-dB steady-state mean-square error degradation, but this could easily be reduced (at the expense of tracking ability) to a negligible amount via gearshifting. The arrangements of either only a decision feedback equalizer (no linear taps) or only an echo canceller alone are both contained in our results; simply set either $L = 0$ or $J = 0$.

The absence of multiplications and A/D converters in both the canceller and the decision feedback equalizer will make implementation attractive. However, despite all the mentioned advantages, both

* This would, for example, be required in DDS extension service.

in regard to economics and convergence properties, it must be realized that there are many channels where compensation of the postcursor ISI is not sufficient. The equalizer will then require linear taps (the canceller, where the bulk of the taps will be concentrated for voiceband data applications, will fortunately never require linear taps). The inclusion of only a few linear taps drastically changes the joint convergence behavior, and more investigation is needed to determine economic architectures and algorithms which, under these conditions, would essentially retain the independence characteristics of the structure shown in Fig. 1.

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